

ATUL VIDYALAYA
SECOND PRELIMINARY EXAMINATION-2012-13
MATHEMATICS

STD: XII Science

DATE: / /

MM: 100

TIME: 3 hrs

GENERAL INSTRUCTION

(Three hours)

(Candidate are allowed additional 15 minutes for **only** reading the paper .
They must **NOT** start writing during this time)

There will be one paper of **three** hours duration of 100 marks. The syllabus is divided into three sections A , B and C. Section A is compulsory for all candidates. Candidates will have choice of attempting questions from **either** from Section B or Section C.

Section A(80 marks) will consists of 9 questions. Candidate will be required to answer **Question -1 (Compulsory)** and **five** out of the rest of the eight question.

Section B/C(20 marks) Candidate will be required to answer two questions out of three from either Section B or Section C.

Section – A

Question 1.

i) Find the value of x , if $\sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{7}{25}\right) = \cot^{-1} x$.

[3]

ii) Find the probability of getting an even number on the first die or a total of 8 in a single throw of two dice . [3]

iii) For the matrix $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y so that $A^2 = xI = yA$. Hence , find A^{-1} .

[3]

iv) Simplify the Boolean expression : $x + x.y' + x'.y$

[3]

v) Evaluate : $\lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x)$

[3]

vi) Show that the equation $y = be^x + ce^{2x}$ is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0 \quad [3]$$

vii) Find the area between the curve $x = y^2 - 2y$, the y – axis and the abscissa , $y = 1$ and $y = 2$. [3]

viii) Evaluate : $\int_0^3 \frac{x}{1+x^2} dx$ [3]

ix) Find the modulus and amplitude of $\frac{2+3i}{3+2i}$. [3]

x) Find " c " of the Lagrange's Mean Value Theorem when $f(x) = x(x-2)$, in $[1, 2]$. [3]

Question 2.

i) Prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$. [5]

- ii) If $A = \begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix}$, find A^{-1} . Using A^{-1} , solve the system of linear equations :
- $$4x - 5y - 11z = 12$$
- $$x - 3y + z = 1$$
- $$2x + 3y - 7z = 2$$

[5]

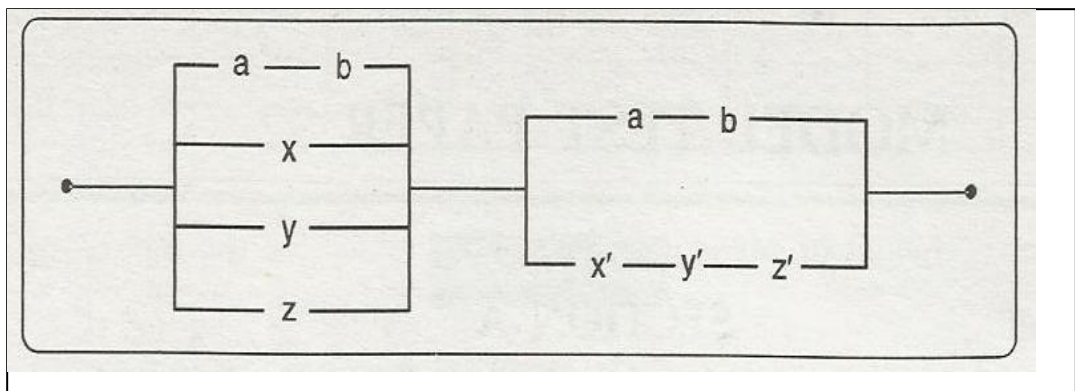
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Question 3.

- i) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$. [5]
- ii) Write the Boolean expression for the following switching circuit :



Using laws of Boolean algebra, simplify the circuit and construct an equivalent switching circuit. [5]

Question 4

- i) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ [5]
- ii) Find the co-ordinate of the points on the ellipse $4x^2 + y^2 = 8$ at which tangents are parallel to the straight line $2x - y = 7$. [5]

Question 5.

- i) Suppose A and B are two equally strong table tennis players. Which of the following two events is more probable ?
- a) A beats b in exactly 3 games out of 4, or
- b) A beats B in exactly 5 games out of 8 ? [5]
- ii) Find the probability of throwing 11 with three dice. [5]

Question 6.

- i) Evaluate : $\int x^2 \sin^{-1} x dx$ [5]
- ii) Prove that the triangle of maximum area inscribed in a given circle must be equilateral. [5]

Question 7.

- i) Find the area of the region lying in the first quadrant bounded by the parabola $y^2 = 4x$, the x-axis and the ordinate $x = 4$. [5]
- ii) Verify Lagrange's Mean Value theorem for the given function in the given interval

and find the value of " c " of this theorem $f(x) = \sqrt{x^2 - 1}$ in $[1, 3]$. [5]

Question 8.

i) Calculate the coefficient of correlation between x and y for the following data : [5]

x	10	7	12	9	16	18	8	14
y	6	4	7	10	7	15	5	11

ii) If two coefficient of regression are -0.6 and -1.4 , find the coefficient of correlation .

Question 9

i) If $1, \omega, \omega^2$ are the three cube roots of unity, show that

$$x^3 - y^3 = (x - y)(\omega x - \omega^2 y)(\omega^2 x - \omega y) \quad [5]$$

ii) Solve : $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$ [5]

(Answer two questions from either Section B or Section C)

SECTION B

Question 10

i) Prove that the triangle whose vertices have the position vectors

$$2\hat{i} + 4\hat{j} - \hat{k}, 4\hat{i} + 5\hat{j} + \hat{k}, 3\hat{i} + 6\hat{j} - 3\hat{k} \text{ is an isosceles right angled triangle .} \quad [5]$$

ii) If $a = 4\hat{i} + 3\hat{j} - \hat{k}, b = 2\hat{i} + \hat{j} + 3\hat{k}$ and $c = \hat{i} - \hat{j} + 2\hat{k}$, then simplify $(a - b) \times (b + c)$. [5]

Question 11

i) A variable plane is at a constant distance p from origin and meets the axes in A, B and C. Through A, B, C planes are drawn parallel to the co-ordinate planes,

$$\text{show that the locus of their point of intersection is } x^{-2} + y^{-2} + z^{-2} = p^{-2}. \quad [5]$$

ii) Find the equation to the line that intersects the lines $2x + y - 1 = 0, x + 2y + 3z = 0$ and

$$3x - y + z + 2 = 0, 4x + 5y - 2z - 3 = 0 \text{ and parallel to the line } \frac{x}{1} = \frac{y}{2} = \frac{z}{3}. \quad [5]$$

Question 12

i) Urn - 1 contains 5 red and 5 black balls, urn - 2 contains 4 red and 8 black balls and urn -3 contains 3 red and 6 black balls. One urn is chosen at random and a ball is drawn. The colour of the ball is black. What is the probability that this has been drawn from urn - 3? [5]

ii) Mean and standard deviation of a binomial distribution are respectively 4 and $\sqrt{\frac{8}{3}}$. Find the value of n and p. [5]

SECTION C

Question 13

i) What is the actual rate of interest which a banker gets for the money when he discounts a bill legally due in 6 months at 5% p.a.? [5]

ii) A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts,

while it takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts .He earns a profit of ` 2.50 per package on nuts and ` 1 per package on bolts . Form a linear programming problem to maximize his profit , if he operates each machine for at least 12 hours . [5]

Question 14

- i) What sum should be invested every year at 5% per annum compound interest for 20 years to replace plant and machinery which is expected to cost then , 25% more , over its present cost of ` 60 , 000 ? [5]

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- ii) Given the cost function $C(x) = 50 + 2x$ and demand function $p = 100 - 4x$.Find the
 (a) equilibrium output
 (b) Maximum price [5]

Question 15

- i) From the following data , giving the prices in arbitrary units and quantities of commodities A , B and C compute the weighted aggregate index of 1992 using 1986 as base year : [5]

Commodity	Quantities	Prices	
		1986	1992
A	7	321	581
B	8	54	67
C	4	224	305

- ii) The following mid-day temperature in degree centigrades were recorded at a place in the U.K. for the first 14 days of January 1975 :

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Tem.	0	4	2	6	2	-1	-3	1	5	0	4	5	6	7

Calculate 5 day moving averages for this period and display them and the original table on the same graph . [5]

